

Single Valued Neutrosophic Graphs: Degree, Order and Size

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Abstract—The single valued neutrosophic graph is a new version of graph theory presented recently as a generalization of fuzzy graph and intuitionistic fuzzy graph. The single valued neutrosophic graph (SVN-graph) is used when the relation between nodes (or vertices) in problems are indeterminate. In this paper, we examine the properties of various types of degrees, order and size of single valued neutrosophic graphs and a new definition for regular single valued neutrosophic graph is given.

Keywords— *single valued neutrosophic graph; total degree; effective degree; neighborhood degree; Order; Size.*

I. INTRODUCTION

Neutrosophic set (NS for short) proposed by Smarandache [11, 12] is a powerful tool to deal with incomplete, indeterminate and inconsistent information in real world. It is a generalization of the theory of fuzzy set [16], intuitionistic fuzzy sets [22, 24], interval-valued fuzzy sets [18] and interval-valued intuitionistic fuzzy sets [23], then the neutrosophic set is characterized by a truth-membership degree (t), an indeterminacy-membership degree (i) and a falsity-membership degree (f) independently, which are within the real standard or nonstandard unit interval $]0, 1[$. Therefore, if their range is restrained within the real standard unit interval $[0, 1]$, Nevertheless, NSs are hard to be apply in practical problems since the values of the functions of truth, indeterminacy and falsity lie in $]0, 1[$. Therefore, Wang et al.[14] presented single-valued neutrosophic sets (SVNSs) whose functions of truth, indeterminacy and falsity lie in $[0, 1]$. The same authors introduced the notion of interval valued neutrosophic sets [15] as subclass of neutrosophic sets in which the value of truth-membership, indeterminacy-membership and falsity-membership degrees are intervals of numbers instead of the real numbers. neutrosophic sets and its extensions such as single valued neutrosophic sets, interval neutrosophic sets, simplified neutrosophic sets and so on have been applied in a wide variety of fields including computer science, engineering, mathematics, medicine and economic [1, 2, 3, 7, 8, 10, 11, 12, 13, 17, 19, 20, 21, 27, 33, 34, 35].

Many works on fuzzy graphs and intuitionistic fuzzy graphs [4, 5, 6, 27, 28, 41] have been carried out and all of them have considered the vertex sets and edge sets as fuzzy and /or intuitionistic fuzzy sets. But, when the relations between nodes (or vertices) in problems are indeterminate, the fuzzy graphs and intuitionistic fuzzy graphs are failed. For this purpose, Smarandache [9] have defined four main categories of neutrosophic graphs, two based on literal indeterminacy (I), which called them; I-edge neutrosophic graph and I-vertex neutrosophic graph, these concepts are studied deeply and has gained popularity among the researchers due to its applications via real world problems [38, 39, 40]. The two others graphs are based on (t, i, f) components and called them; The (t, i, f) -edge neutrosophic graph and the (t, i, f) -vertex neutrosophic graph, these concepts are not developed at all. Later on, Broumi et al. [30] introduced a third neutrosophic graph model combined the (t, i, f) -edge and the (t, i, f) -vertex neutrosophic graph and investigated some of their properties. The third neutrosophic graph model is called ‘single valued neutrosophic graph’ (SVNG for short). The single valued neutrosophic graph is the generalization of fuzzy graph and intuitionistic fuzzy graph. Also, Broumi et al.[31] introduced the concept of bipolar single valued neutrosophic graph as a generalization of fuzzy graphs, intuitionistic fuzzy graph, N-graph, bipolar fuzzy graph and single valued neutrosophic graph and studied some of their related properties. The same authors [32, 33, 34], introduced the concept of interval valued neutrosophic graph as a generalization of single valued neutrosophic graph and have discussed some of their properties with proof and examples. The remainder of this paper is organized as follows. In Section 2, we review some basic concepts about neutrosophic sets, single valued neutrosophic sets, single valued neutrosophic graph and complete single valued neutrosophic graph. The type of degrees in single valued neutrosophic graphs such as degree of vertex, total degree, effective degree, neighborhood degree, closed neighborhood degree are defined in Section 3. Furthermore, some properties of the proposed degrees are discussed with numerical examples. In Section 4, we present

the concept of regular single valued neutrosophic graph and proved some propositions. In addition, Section 5 also present the concept of order and size of single valued neutrosophic graph. Finally, Section 6 outlines the conclusion of this paper and suggests several directions for future research.

II. PRELIMINARIES

In this section, we mainly recall some notions related to neutrosophic sets, single valued neutrosophic sets, fuzzy graph, intuitionistic fuzzy graph, single valued neutrosophic graphs, relevant to the present work. See especially [12, 14, 26, 28] for further details and background.

Definition 2.1 [12]. Let X be a space of points (objects) with generic elements in X denoted by x ; then the neutrosophic set A (NS A) is an object having the form $A = \{x: T_A(x), I_A(x), F_A(x) >, x \in X\}$, where the functions T , I , F : $X \rightarrow [0,1]^+$ [define respectively the truth-membership function, an indeterminacy-membership function, and a falsity-membership function of the element $x \in X$ to the set A with the condition:

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+. \quad (1)$$

The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $[0,1]^+$.

Since it is difficult to apply NSs to practical problems, Wang et al. [14] introduced the concept of a SVNS, which is an instance of a NS and can be used in real scientific and engineering applications.

Definition 2.2 [14]. Let X be a space of points (objects) with generic elements in X denoted by x . A single valued neutrosophic set A (SVNS A) is characterized by truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. For each point x in X $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$. A SVNS A can be written as

$$A = \{x: T_A(x), I_A(x), F_A(x) >, x \in X\} \quad (2)$$

Definition 2.3[7]. A fuzzy graph is a pair of functions $G = (\sigma, \mu)$ where σ is a fuzzy subset of a non empty set V and μ is a symmetric fuzzy relation on σ . i.e $\sigma : V \rightarrow [0,1]$ and $\mu : V \times V \rightarrow [0,1]$ such that $\mu(uv) \leq \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$ where uv denotes the edge between u and v and $\sigma(u) \wedge \sigma(v)$ denotes the minimum of $\sigma(u)$ and $\sigma(v)$. σ is called the fuzzy vertex set of V and μ is called the fuzzy edge set of E .

Definition 2.4 [26]: An intuitionistic fuzzy graph (IFG) is of the form $G = (V, E)$ where

$1. V = \{v_1, v_2, \dots, v_n\}$ such that $\mu_1 : V \rightarrow [0, 1]$ and $\gamma_1 : V \rightarrow [0, 1]$ denotes the degree of membership and non-membership of the element $v_i \in V$, respectively, and

$$0 \leq \mu_1(v_i) + \gamma_1(v_i) \leq 1 \text{ for every } v_i \in V \text{ (i=1, 2, ..., n)} \quad (3)$$

$2. E \subseteq V \times V$ where $\mu_2 : V \times V \rightarrow [0, 1]$ and $\gamma_2 : V \times V \rightarrow [0, 1]$ are such that

$$\mu_2(v_i, v_j) \leq \min[\mu_1(v_i), \mu_1(v_j)] \quad (4)$$

$$\gamma_2(v_i, v_j) \geq \max[\gamma_1(v_i), \gamma_1(v_j)] \quad (5)$$

$$0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 1 \text{ for every } (v_i, v_j) \in E \text{ (i, j= 1, 2, ..., n)} \quad (6)$$

Definition 2.4 [28]. A single valued neutrosophic graph (SVN-graph) with underlying set V is defined to be a pair $G = (A, B)$ where

1.The functions $T_A : V \rightarrow [0, 1]$, $I_A : V \rightarrow [0, 1]$ and $F_A : V \rightarrow [0, 1]$ denote the degree of truth-membership, degree of indeterminacy-membership and falsity-membership of the element $v_i \in V$, respectively, and

$$0 \leq T_A(v_i) + I_A(v_i) + F_A(v_i) \leq 3 \text{ for all } v_i \in V.$$

2. The functions $T_B : E \subseteq V \times V \rightarrow [0, 1]$, $I_B : E \subseteq V \times V \rightarrow [0, 1]$ and $F_B : E \subseteq V \times V \rightarrow [0, 1]$ are defined by

$$F_B(v_i, v_j) \leq \min [T_A(v_i), T_A(v_j)], I_B(v_i, v_j) \geq \max [I_A(v_i), I_A(v_j)] \text{ and } F_B(v_i, v_j) \geq \max [F_A(v_i), F_A(v_j)]$$

Denotes the degree of truth-membership, indeterminacy-membership and falsity-membership of the edge $(v_i, v_j) \in E$ respectively, where

$$0 \leq T_B(v_i, v_j) + I_B(v_i, v_j) + F_B(v_i, v_j) \leq 3 \text{ for all } (v_i, v_j) \in E \text{ (i, j= 1, 2, ..., n)}$$

We call A the single valued neutrosophic vertex set of V , B the single valued neutrosophic edge set of E , respectively.

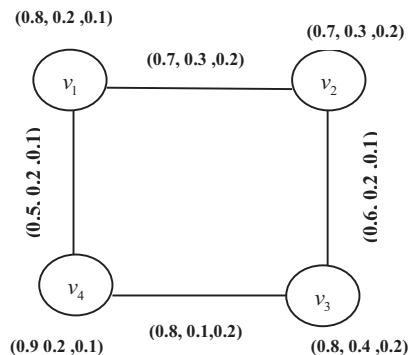


Fig.1. Single valued neutrosophic graph

Definition 2.5 [28]. A partial SVN-subgraph of SVN-graph $G = (A, B)$ is a SVN-graph $H = (V', E')$ such that

- $V' \subseteq V$, where $T_A'(v_i) \leq T_A(v_i), I_A'(v_i) \geq I_A(v_i)$, and $F_A'(v_i) \geq F_B(v_i)$, for all $v_i \in V$.

- $E' \subseteq E$, where $T_B'(v_i, v_j) \leq T_B(v_i, v_j), I_B'(v_i, v_j) \geq I_B(v_i, v_j)$, and $F_B'(v_i, v_j) \geq F_B(v_i, v_j)$, for all $(v_i, v_j) \in E$.

Definition 2.6[28]. A single valued neutrosophic graph $G = (A, B)$ of $G^* = (V, E)$ is called complete single valued neutrosophic graph if

$$T_B(v_i, v_j) = \min [T_A(v_i), T_A(v_j)]$$

$$I_B(v_i, v_j) = \max [I_A(v_i), I_A(v_j)]$$

$$F_B(v_i, v_j) = \max [F_A(v_i), F_A(v_j)] \text{ for all } v_i, v_j \in V.$$

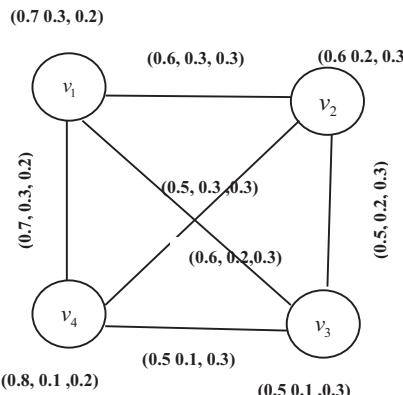


FIG. 2. COMPLETE SINGLE VALUED NEUTROSOPHIC GRAPH

III. TYPE OF DEGREES IN SINGLE VALUED NEUTROSOPHIC GRAPHS

In this section, degree of vertex, total degree, effective degree, neighbourhood degree, closed neighbourhood degree are introduced.

Definition 3.1: Let $G = (A, B)$ be a single valued neutrosophic graph. Then the degree of a vertex $v_i \in G$ is sum of degree of truth-membership, sum of degree of indeterminacy-membership and sum of degree of falsity-membership of all those edges which are incident on vertex v denoted by

$$d(v_i) = (d_T(v_i), d_I(v_i), d_F(v_i)) \text{ where}$$

$d_T(v_i) = \sum_{v_i \neq v_j} T_B(v_i, v_j)$ denotes degree of truth-membership vertex.

$d_I(v_i) = \sum_{v_i \neq v_j} I_B(v_i, v_j)$ denotes degree of indeterminacy-membership vertex.

$d_F(v_i) = \sum_{v_i \neq v_j} F_B(v_i, v_j)$ denotes degree of falsity-membership vertex, for $v_i, v_j \in A$ and $T_B(v_i, v_j) = 0, I_B(v_i, v_j) = 0, F_B(v_i, v_j) = 0$ for $v_i, v_j \notin A$.

Definition 3.2: Let $G = (A, B)$ be a single valued neutrosophic graph. Then the total degree of a vertex $v_i \in G$ is defined by

$$td(v_i) = (td_T(v_i), td_I(v_i), td_F(v_i)) \text{ where}$$

$td_T(v_i) = \sum_{v_i \neq v_j} T_B(v_i, v_j) + T_A(v_i)$ denotes total degree of truth-membership vertex.

$td_I(v_i) = \sum_{v_i \neq v_j} I_B(v_i, v_j) + I_A(v_i)$ denotes total degree of indeterminacy-membership vertex.

$td_F(v_i) = \sum_{v_i \neq v_j} F_B(v_i, v_j) + F_A(v_i)$ denotes total degree of falsity-membership vertex. for $v_i, v_j \in A$.

Definition 3.2: The minimum degree of G is $\delta(G) = (\delta_T(G), \delta_I(G), \delta_F(G))$, where

$\delta_T(G) = \wedge \{ d_T(v) | v \in V \}$ denotes the minimum T-degree.

$\delta_I(G) = \wedge \{ d_I(v) | v \in V \}$ denotes the minimum I-degree.

$\delta_F(G) = \wedge \{ d_F(v) | v \in V \}$ denotes the minimum F-degree.

Definition 3.3: The maximum degree of G is $\Delta(G) = (\Delta_T(G), \Delta_I(G), \Delta_F(G))$, where

$\Delta_T(G) = \vee \{ d_T(v) | v \in V \}$ denotes the maximum T-degree.

$\Delta_I(G) = \vee \{ d_I(v) | v \in V \}$ denotes the maximum I-degree.

$\Delta_F(G) = \vee \{ d_F(v) | v \in V \}$ denotes the maximum F-degree.

Example 3.4: Consider a SVN-graph $G = (V, E)$, such that $V = \{v_1, v_2, v_3, v_4\}$ and $E = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_1)\}$

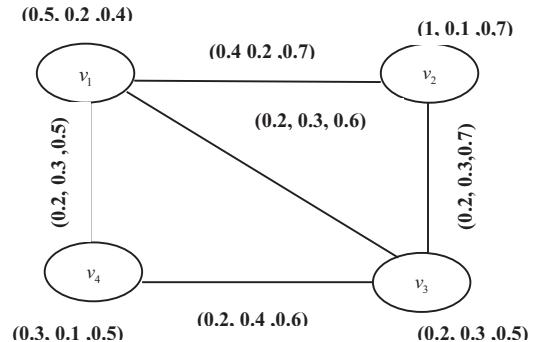


Fig. 3. Single valued neutrosophic graph

By usual computation, we have

$$\begin{aligned} d(v_1) &= (0.8, 0.8, 1.8), \quad d(v_2) = (0.6, 0.5, 1.4) \\ d(v_3) &= (0.6, 1, 1.9), \quad d(v_4) = (0.4, 0.7, 1.1) \\ td(v_1) &= (1.3, 1, 2.2), \quad td(v_2) = (1.6, 0.6, 2.1) \\ td(v_3) &= (0.8, 1.3, 1.3), \quad td(v_4) = (0.7, 0.8, 1.6) \text{ and} \\ \delta(G) &= (0.4, 0.5, 1.1) \quad \Delta(G) = (0.8, 1, 1.9). \end{aligned}$$

Proposition 3.5: In any single valued neutrosophic graph $G=(V, E)$, the sum of the degree of truth-membership value of all vertices is equal to twice the sum of the truth-membership value of all edges, the sum of the degree of indeterminacy-membership value of all vertices is equal to twice the sum of the indeterminacy-membership value of all edges and the sum of the degree of falsity-membership value of all vertices is equal to twice the sum of the falsity-membership value of all edges.

$$\begin{aligned} \sum d(v_i) &= [\sum d_T(v_i), \sum d_I(v_i), \sum d_F(v_i)] \\ &= [2 \sum_{v_i \neq v_j} T_B(v_i, v_j), 2 \sum_{v_i \neq v_j} I_B(v_i, v_j), \\ &\quad 2 \sum_{v_i \neq v_j} F_B(v_i, v_j)]. \end{aligned} \quad (9)$$

Let $G = (A, B)$ be a SVN-graph where $V=\{v_1, v_2, \dots, v_n\}$

$$\begin{aligned} \sum d(v_i) &= [\sum d_T(v_i), \sum d_I(v_i), \sum d_F(v_i)] = \\ &= [(d_T(v_1), d_I(v_1), d_F(v_1)) + (d_T(v_2), d_I(v_2), d_F(v_2)) + \\ &\quad + \dots + (d_T(v_n), d_I(v_n), d_F(v_n))] \\ &= [(T_B(v_1, v_2), I_B(v_1, v_2), F_B(v_1, v_2)) + (T_B(v_1, v_3), \\ &\quad I_B(v_1, v_3), F_B(v_1, v_3)) + \dots + (T_B(v_1, v_n), I_B(v_1, v_n), \\ &\quad F_B(v_1, v_n)) + (T_B(v_2, v_1), I_B(v_2, v_1), F_B(v_2, v_1)) + \\ &\quad (T_B(v_2, v_3), I_B(v_2, v_3), F_B(v_2, v_3)) + \dots + (T_B(v_2, v_n), \\ &\quad I_B(v_2, v_n), F_B(v_2, v_n)) + \\ &\quad \dots + (T_B(v_n, v_1), I_B(v_n, v_1), F_B(v_n, v_1)) + (T_B(v_n, v_2), \\ &\quad I_B(v_n, v_2), F_B(v_n, v_2)) + \dots + (T_B(v_{n-1}, v_n), I_B(v_{n-1}, v_n), \\ &\quad F_B(v_{n-1}, v_n))] \\ &= [2[(T_B(v_1, v_2), I_B(v_1, v_2), F_B(v_1, v_2)) + (T_B(v_1, v_3), \\ &\quad I_B(v_1, v_3), F_B(v_1, v_3)) + \dots + ((T_B(v_1, v_n), I_B(v_1, v_n), \\ &\quad F_B(v_1, v_n)))] \\ &= [2 \sum_{v_i \neq v_j} T_B(v_i, v_j), 2 \sum_{v_i \neq v_j} I_B(v_i, v_j), \\ &\quad 2 \sum_{v_i \neq v_j} F_B(v_i, v_j)]. \text{ Hence the proof.} \end{aligned}$$

Proposition 3.6: The maximum degree of any vertex in a SVN-graph with n vertices is $n-1$.

Proof: Let $G = (A, B)$ be a SVN-graph. The maximum truth-membership value given to an edge is 1 and the number of edges incident on a vertex can be at most $n-1$. Hence, the maximum truth-membership degree $d_T(v_i)$ of any vertex v_i in a SVN-graph with n vertices is $n-1$. Similarly, the maximum indeterminacy-membership value given to an edge is 1 and the number of edges incident on a vertex can be at most $n-1$. Hence the maximum indeterminacy-membership degree $d_I(v_i)$. Also, the maximum falsity-membership value given to an edge is 1 and the number of edges incident on a vertex can be at most $n-1$. Hence the maximum falsity-membership degree $d_F(v_i)$ of any vertex v_i in a SVN-graph with n vertices is $n-1$. Hence the result.

Definition 3.7: An edge $e = (v, w)$ of a SVN-graph $G = (A, B)$ is called an effective edge if $T_B(v, w) = T_A(v) \wedge T_A(w)$, $I_B(v, w) = I_A(v) \vee I_A(w)$ and $F_B(v, w) = F_A(v) \vee F_A(w)$ for all $(v, w) \in E$. In this case, the vertex v is called a neighbor of w and conversely.

$N(v) = \{w \in V : w \text{ is a neighbor of } v\}$ is called the neighborhood of v .

Example 3.8. Consider a SVN-graph $G = (A, B)$, such that $A = \{v_1, v_2, v_3, v_4\}$ and $B = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_1)\}$ $(0.8, 0.2, 0.1)$ $(0.7, 0.3, 0.2)$ $(0.6, 0.4, 0.3)$ $(0.9, 0.1, 0.1)$ $(0.8, 0.4, 0.2)$

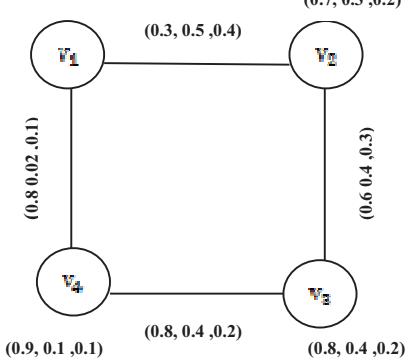


Fig. 4. Single valued neutrosophic graph

In this example, v_4 - v_1 and v_4 - v_3 are effective edges. Also $N(v_4) = \{v_1, v_3\}$, $N(v_3) = \{v_4\}$, $N(v_1) = \{v_4\}$, $N(v_2) = \emptyset$ (the empty set).

Definition 3.9: The effective degree of a vertex 'v' in G is defined by $d_E(v) = (d_{ET}(v), d_{EI}(v), d_{EF}(v))$, where $d_{ET}(v)$ is the sum of the truth-membership values of the effective edges incident with v , $d_{EI}(v)$ is the sum of the indeterminacy-membership values of the effective edges

incident with v and $d_{EF}(v)$ is the sum of the falsity-membership values of effective edges incident with v .

Definition 3.10: The minimum effective degree of G is $\delta_E[G] = (\delta_{ET}[G], \delta_{EI}[G], \delta_{EF}[G])$ where

$\delta_{ET}[G] = \wedge \{ d_{ET}[v] | v \in V \}$ denotes the minimum effective T-degree.

$\delta_{EI}[G] = \wedge \{ d_{EI}[v] | v \in V \}$ denotes the minimum effective I-degree.

$\delta_{EF}[G] = \wedge \{ d_{EF}[v] | v \in V \}$ denotes the minimum effective F-degree.

Definition 3.11: The maximum effective degree of G is $\Delta_E[G] = (\Delta_{ET}[G], \Delta_{EI}[G], \Delta_{EF}[G])$ where

$\Delta_{ET}[G] = \vee \{ d_{ET}[v] | v \in V \}$ denotes the maximum effective T-degree.

$\Delta_{EI}[G] = \vee \{ d_{EI}[v] | v \in V \}$ denotes the maximum effective I-degree.

$\Delta_{EF}[G] = \vee \{ d_{EF}[v] | v \in V \}$ denotes the maximum effective F-degree.

Example 3.12: Consider a SVN-graph as in Fig.3. By usual computation, we have the effective degrees for all vertices

$$d_E(v_1) = (0, 0, 0) \quad d_E(v_2) = (0.2, 0.3, 0.7),$$

$$d_E(v_3) = (0.2, 0.3, 0.7) \quad d_E(v_4) = (0, 0, 0)$$

$$\delta_E(G) = (0, 0, 0) \quad \Delta_E(G) = (0.2, 0.3, 0.7)$$

Here v_2, v_3 is only effective degree.

Note: $d_E(v_1) = (0, 0, 0)$ means that there is no effective edge incident on v_1 .

Now, we can defined the neighborhood concept in SVN-graph.

Definition 3.13: Let $G = (A, B)$ be a SVN-graph. The neighbourhood of any vertex v is defined as $N(v) = (N_T(v), N_I(v), N_F(v))$, where

$N_T(v) = \{ T_B(v, w) = T_A(v) \wedge T_A(w) ; w \in V \}$ denotes the neighbourhood T-vertex.

$N_I(v) = \{ I_B(v, w) = I_A(v) \vee I_A(w) ; w \in V \}$ denotes the neighbourhood I-vertex.

$N_F(v) = \{ F_B(v, w) = F_A(v) \vee F_A(w) ; w \in V \}$ denotes the neighbourhood F-vertex.

And $N[v] = N(v) \cup \{v\}$ is called the closed neighbourhood of v .

Definition 3.14: Let $G = (A, B)$ be a single valued neutrosophic graph (SVN-graph). The neighbourhood degree of a vertex ' v ' is defined as the sum of truth-membership, indeterminacy-

membership and falsity-membership value of the neighbourhood vertices of v and is denoted by

$$d_N(v) = (d_{NT}(v), d_{NI}(v), d_{NF}(v)), \text{where}$$

$d_{NT}(v) = \sum_{w \in N(v)} T_A(w)$ denotes the neighbourhood T-degree.

$d_{NI}(v) = \sum_{w \in N(v)} I_A(w)$ denotes the neighbourhood I-degree.

$d_{NF}(v) = \sum_{w \in N(v)} F_A(w)$ denotes neighbourhood F-degree.

Definition 3.15: The minimum neighbourhood degree is defined as

$$\delta_N(G) = (\delta_{NT}(G), \delta_{NI}(G), \delta_{NF}(G)), \text{where}$$

$\delta_{NT}(G) = \wedge \{ d_{NT}(v) | v \in V \}$ denotes the minimum neighbourhood T-degree.

$\delta_{NI}(G) = \wedge \{ d_{NI}(v) | v \in V \}$ denotes the minimum neighbourhood I-degree.

$\delta_{NF}(G) = \wedge \{ d_{NF}(v) | v \in V \}$ denotes the minimum neighbourhood F-degree.

Definition 3.16: The maximum neighbourhood degree is defined as

$$\Delta_N(G) = (\Delta_{NT}(G), \Delta_{NI}(G), \Delta_{NF}(G)) \text{ where}$$

$\Delta_{NT}(G) = \vee \{ d_{NT}(v) | v \in V \}$ denotes the maximum neighbourhood T-degree.

$\Delta_{NI}(G) = \vee \{ d_{NI}(v) | v \in V \}$ denotes the maximum neighbourhood I-degree.

$\Delta_{NF}(G) = \vee \{ d_{NF}(v) | v \in V \}$ denotes the maximum neighbourhood F-degree.

Example 3.17: Consider a SVN-graph as in Fig. 2. By usual computation, we have the neighbourhood degrees for all vertices, minimum and maximum neighbourhood degrees

$$d_N(v_1) = (1.9, 0.4, 0.8) \quad d_N(v_2) = (2, 0.5, 0.7)$$

$$d_N(v_3) = (2.1, 0.6, 0.7) \quad d_N(v_4) = (1.8, 0.6, 0.8)$$

$$\delta_N(G) = (1.8, 0.4, 0.7) \quad \Delta_N(G) = (2.1, 0.6, 0.8).$$

Definition 3.18: A vertex $v \in V$ of SVN-graph $G = (A, B)$ is said to be an isolated vertex if $T_B(v_i, v_j) = I_B(v_i, v_j) = F_B(v_i, v_j) = 0$ For all $v \in V$, $v_i \neq v_j$ that is $N(v) = \emptyset$

Definition 3.19: Let $G = (A, B)$ be a single valued neutrosophic graph (SVN-graph). The closed neighbourhood degree of a vertex ' v ' is defined as the sum of truth-membership, indeterminacy-membership and falsity-membership value of the neighbourhood vertices of v and including truth-membership, indeterminacy-membership and

falsity-membership value of v , and is denoted by $d_N[v] = (d_{NT}[v], d_{NI}[v], d_{NF}[v])$ where

$d_{NT}[v] = \sum_{w \in N(v)} T_A(w) + T_A(v)$ denotes the closed neighborhood T-degree.

$d_{NI}[v] = \sum_{w \in N(v)} I_A(w) + I_A(v)$ denotes the closed neighborhood I-degree.

$d_{NF}[v] = \sum_{w \in N(v)} F_A(w) + F_A(v)$ denotes the closed neighborhood F-degree.

Definition 3.20: The minimum closed neighborhood degree is defined as

$$\delta_N[G] = (\delta_{NT}[G], \delta_{NI}[G], \delta_{NF}[G]) \text{ where}$$

$\delta_{NT}[G] = \wedge \{ d_{NT}[v] | v \in V \}$ denotes the minimum closed neighborhood T-degree

$\delta_{NI}[G] = \wedge \{ d_{NI}[v] | v \in V \}$ denotes the minimum closed neighborhood I-degree

$\delta_{NF}[G] = \wedge \{ d_{NF}[v] | v \in V \}$ denotes the minimum closed neighborhood F-degree

Definition 3.21: The maximum closed neighborhood degree is defined as

$$\Delta_N[G] = (\Delta_{NT}[G], \Delta_{NI}[G], \Delta_{NF}[G]) \text{ where}$$

$\Delta_{NT}[G] = \vee \{ d_{NT}[v] | v \in V \}$ denotes the maximum closed neighborhood T-degree

$\Delta_{NI}[G] = \vee \{ d_{NI}[v] | v \in V \}$ denotes the maximum closed neighborhood I-degree

$\Delta_{NF}[G] = \vee \{ d_{NF}[v] | v \in V \}$ denotes the maximum closed neighborhood F-degree.

IV. REGULAR SINGLE VALUED NEUTROSOPHIC GRAPH.

Definition 4.1: A single valued neutrosophic graph $G = (A, B)$ is said to be regular single valued neutrosophic graph (RSVN-graph), if all the vertices have the same closed neighborhood degree. (i.e) if $\delta_{NT}[G] = \Delta_{NT}[G]$, $\delta_{NI}[G] = \Delta_{NI}[G]$ and $\delta_{NF}[G] = \Delta_{NF}[G]$

Example 4.2: Consider a SVN-graph as in Fig. 2. By usual computation, we have the closed neighborhood degrees for all vertices, minimum and maximum neighborhood degrees

$$d_N[v_1] = d_N[v_2] = d_N[v_3] = d_N[v_4] = (2.6, 0.7, 1)$$

$$\delta_N[G] = \Delta_N[G] = (2.6, 0.7, 1)$$

It is clear from calculation that G is regular single valued neutrosophic graph (RSVN-graph).

Theorem 4.3: Every complete single valued neutrosophic is a regular single valued neutrosophic graph

Proof:

Let $G = (A, B)$ be a complete SVN-graph then by definition of complete SVN-graph we have

$$T_B(v, w) = T_A(v) \wedge T_A(w), I_B(v, w) = I_A(v) \vee I_A(w), F_B(v, w) = F_A(v) \vee F_A(w), \text{ for every } v, w \in V.$$

By definition, the closed neighborhood T-degree of each vertex is the sum of the truth-membership values of the vertices and itself, the closed neighborhood I-degree of each vertex is the sum of the indeterminacy-membership values of the vertices and itself and the closed neighborhood F-degree of each vertex is the sum of the falsity-membership values of the vertices and itself, Therefore all the vertices will have the same closed neighborhood T-degree, closed neighborhood -degree and closed neighborhood F -degree. This implies minimum closed neighborhood degree is equal to maximum closed neighborhood degree (i.e) $\delta_{NT}[G] = \Delta_{NT}[G]$, $\delta_{NI}[G] = \Delta_{NI}[G]$ and $\delta_{NF}[G] = \Delta_{NF}[G]$. This implies G is a regular single valued neutrosophic graph. Hence the theorem.

V. ORDER AND SIZE OF SINGLE VALUED NEUTROSOPHIC GRAPH

In this section we introduce the definition of order and size of a single valued neutrosophic graph which are an important terms in single valued neutrosophic graph theory.

Definition 5.1: Let $G = (A, B)$ be a SVN-graph. The order of G , denoted $O(G)$ is defined as $O(G) = (O_T(G), O_I(G), O_F(G))$, where

$$O_T(G) = \sum_{v \in V} T_A \text{ denotes the T-order of } G.$$

$$O_I(G) = \sum_{v \in V} I_A \text{ denotes the I-order of } G.$$

$$O_F(G) = \sum_{v \in V} F_A \text{ denotes the F-order of } G.$$

Definition 5.2: Let $G = (A, B)$ be a SVN-graph. The size of G , denoted $S(G)$ is defined as:

$$S(G) = (S_T(G), S_I(G), S_F(G)), \text{ where}$$

$$S_T(G) = \sum_{u \neq v} T_B(u, v) \text{ denotes the T-size of } G.$$

$$S_I(G) = \sum_{u \neq v} I_B(u, v) \text{ denotes the I-size of } G$$

$$S_F(G) = \sum_{u \neq v} F_B(u, v) \text{ denotes the F-size of } G$$

Example 5.3: Consider a SVN-graph as in Fig. 3. By routine computation, we have

$$O(G) = (2, 0.7, 2.1), S(G) = (1.2, 1.5, 3.1)$$

Proposition 5.4: In a complete single valued neutrosophic graph $G = (A, B)$, the closed neighbourhood degree of any vertex is equal to the order of single valued neutrosophic graph

(i.e) $O_T(G) = (d_{NT}[v] \mid v \in V)$, $O_I(G) = (d_{NI}[v] \mid v \in V)$ and $O_F(G) = (d_{NF}[v] \mid v \in V)$

Proof:

Let $G = (A, B)$ be a complete single valued neutrosophic graph. The T-order of G , $O_T(G)$ is the sum of the truth-membership value of all the vertices, the I-order of G , $O_I(G)$ is the sum of the indeterminacy-membership value of all the vertices and the F-order of G , $O_F(G)$ is the sum of the falsity-membership value of all the vertices. Since G is a complete SVN-graph, the closed neighborhood T-degree of each vertex is the sum of the truth-membership value of vertices, the closed neighborhood I-degree of each vertex is the sum of the indeterminacy-membership value of vertices and the closed neighborhood F-degree of each vertex is the sum of the falsity-membership value of vertices. Hence the result.

VI. CONCLUSION

In this paper we have described degree of a vertex, total degree, effective degree, neighborhood degree, closed neighborhood, order and size of single valued neutrosophic graphs. The necessary and sufficient conditions for a single valued neutrosophic graph to be the regular single valued neutrosophic graphs have been presented. Further, we are going to study some types of single valued neutrosophic graphs such irregular and totally irregular single valued neutrosophic graphs and single valued neutrosophic hypergraphs.

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REFERENCES

- [1] A. Q. Ansari, R.Biswas & S. Aggarwal, "Neutrosophic classifier: An extension of fuzzy classifier," Elsevier- *Applied Soft Computing*, 13, 2013, pp.563-573.
- [2] A. Q. Ansari, R.Biswas & S. Aggarwal, "Neutrosophication of Fuzzy Models," IEEE Workshop On Computational Intelligence: Theories, Applications and Future Directions (hosted by IIT Kanpur), 14th July'13.
- [3] A. Q. Ansari, R. Biswas & S. Aggarwal," Extension to fuzzy logic representation: Moving towards neutrosophic logic - A new laboratory rat," *Fuzzy Systems (FUZZ)*, 2013 IEEE International Conference, 2013, pp.1 – 8.
- [4] A. Nagoor Gani and M.Basheer Ahamed, "Order and Size in Fuzzy Graphs," *Bulletin of Pure and Applied Sciences*, Vol 22E ,No.1, 2003, pp.145-148.
- [5] A. Nagoor Gani. A and S. Shajitha Begum," Degree, Order and Size in Intuitionistic Fuzzy Graphs," *International Journal of Algorithms, Computing and Mathematics*,(3)3,2010, pp.11-16.
- [6] A. Nagoor Gani and S.R Latha, "On Irregular Fuzzy Graphs," *Applied Mathematical Sciences*, Vol.6, no.11, 2012, pp.517-523.
- [7] A. V. Devadoss, A. Rajkumar & N. J. P. Praveena,"A Study on Miracles through Holy Bible using Neutrosophic Cognitive Maps (NCMS)," *International Journal of Computer Applications*, 69(3), 2013,
- [8] G. Garg, K. Bhutani, M. Kumar and S. Aggarwal, "Hybrid model for medical diagnosis using Neutrosophic Cognitive Maps with Genetic Algorithms," *FUZZ-IEEE*, 2015, 6page.
- [9] F. Smarandache," Refined Literal Indeterminacy and the Multiplication Law of Sub-Indeterminacies," *Neutrosophic Sets and Systems*, Vol. 9, 2015, pp.58-63.
- [10] F. Smarandache, "Types of Neutrosophic Graphs and neutrosophic Algebraic Structures together with their Applications in Technology," seminar, Universitatea Transilvania din Brasov, Facultatea de Design de Produs si Mediu, Brasov, Romania 06 June 2015.
- [11] F. Smarandache, "Neutrosophic set - a generalization of the intuitionistic fuzzy set," *Granular Computing*, 2006 IEEE International Conference, 2006, p. 38 – 42.
- [12] F. Smarandache, "A geometric interpretation of the neutrosophic set — A generalization of the intuitionistic fuzzy set," *Granular Computing (GrC)*, 2011 IEEE International Conference, 2011, pp.602–606
- [13] H .Wang, Y. Zhang, R. Sunderraman, "Truth-value based interval neutrosophic sets," *Granular Computing*, 2005 IEEE International Conference, vol. 1, 2005, pp. 274 – 277.
- [14] H. Wang, F. Smarandache, Y. Zhang, and R. Sunderraman, "Single valued Neutrosophic Sets," *Multisspace and Multistructure* 4, 2010, pp. 410-413.
- [15] H. Wang, F. Smarandache, Zhang, Y.-Q. and R. Sunderraman, "Interval Neutrosophic Sets and Logic: Theory and Applications in Computing," *Hexis*, Phoenix, AZ, 2005.
- [16] H.J. Zimmermann, *Fuzzy Set Theory and its Applications*, Kluwer-Nijhoff, Boston, 1985.
- [17] I. Deli, M. Ali, F. Smarandache, "Bipolar neutrosophic sets and their application based on multi-criteria decision making problems," *Advanced Mechatronic Systems (ICAMechS)*, 2015 International Conference, 2015, pp.249 – 254.
- [18] I. Turksen, "Interval valued fuzzy sets based on normal forms," *Fuzzy Sets and Systems*, vol. 20,1986, pp. 191-210.
- [19] J. Ye, "Vector similarity measures of simplified neutrosophic sets and their application in multicriteria decision making," *International Journal of Fuzzy Systems*, Vol. 16, No. 2, 2014, pp.204-211.
- [20] J. Ye, "Single-Valued Neutrosophic Minimum Spanning Tree and Its Clustering Method," *Journal of Intelligent Systems* 23(3), 2014, pp. 311–324.
- [21] J. Ye, "Another form of correlation coefficient between single valued neutrosophic sets and its multiple attribute decision-making method," *Neutrosophic Sets and Systems*, 1(1), 2013, pp. 8-12.
- [22] K. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 20, 1986, pp. 87-96.
- [23] K. Atanassov and G. Gargov, Interval valued intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, vol.31, 1989, pp. 343-349.
- [24] K. Atanassov, "Intuitionistic fuzzy sets: theory and applications," *Physica*, New York, 1999.
- [25] M. Ali, and F. Smarandache, "Complex Neutrosophic Set," *Neural Computing and Applications*, Vol. 25, 2016, pp.1-18.
- [26] M. Ali, I. Deli, F. Smarandache, "The Theory of Neutrosophic Cubic Sets and Their Applications in Pattern Recognition," *Journal of Intelligent and Fuzzy Systems*, (In press), pp. 1-7.
- [27] P. Bhattacharya," Some remarks on fuzzy graphs," *Pattern Recognition Letters* 6, 1987, pp.297-302.
- [28] R. Parvathi and M. G. Karunambigai, "Intuitionistic Fuzzy Graphs," *Computational Intelligence, Theory and applications*, International Conference in Germany, 2006, pp.18 -20.
- [29] R. Shahin, "Neutrosophic hierarchical clustering algorithms," *Neutrosophic Sets and Systems*, vol 2, 2014, 18-24.
- [30] S. Broumi, M. Talea, A. Bakali, F. Smarandache, "Single Valued Neutrosophic Graphs," *Journal of New Theory*, N 10, 2016, pp. 86-101.
- [31] S. Broumi, M. Talea, A. Bakali, F. Smarandache, "On Bipolar Single Valued Neutrosophic Graphs," *Journal of New Theory*, N11, 2016, pp.84-102.
- [32] S. Broumi, M. Talea, A. Bakali, F. Smarandache, Interval Valued Neutrosophic Graphs, (2016) in press.
- [33] S. Broumi, M. Talea, A.Bakali, F. Smarandache, "Operations on Interval Valued Neutrosophic Graphs," (2016) unpublished.
- [34] S. Broumi, M. Talea, A. Bakali, F. Smarandache, "Strong Interval Valued Neutrosophic Graphs," (2016) unpublished.
- [35] S. Aggarwal, R. Biswas, A. Q. Ansari, "Neutrosophic modeling and control," *Computer and Communication Technology (ICCCCT)*, 2010 International Conference, 2010, pp. 718–723.
- [36] S. Broumi, F. Smarandache, "New distance and similarity measures of interval neutrosophic sets," *Information Fusion (FUSION)*, 2014 IEEE 17th International Conference, 2014, pp. 1 – 7.
- [37] Y. Hai-Long, G. She, Yanhonge, L. Xiuwu, "On single valued neutrosophic relations," *Journal of Intelligent & Fuzzy Systems*, vol. Preprint, no. Preprint, 2015, pp. 1-12.

- [38] W. B. Vasantha Kandasamy and F. Smarandache, “Fuzzy Cognitive Maps and Neutrosophic Cognitive Maps” 2013.
- [39] W.B. Vasantha Kandasamy, K. Ilanthenral and Florentin Smarandache, “Neutrosophic Graphs: A New Dimension to Graph Theory,” Kindle Edition, 2015.
- [40] W.B. Vasantha Kandasamy and F. Smarandache, “Analysis of social aspects of migrant laborers living with HIV/AIDS using Fuzzy Theory and Neutrosophic Cognitive Maps,” Xiquan, Phoenix (2004).
- [41] W.B. Vasantha Kandasamy and R. THANGARAJ, “Vertex-edge Matrix of fuzzy Graphs,” Ultra Scientist Vol. 27(1)A, 2015,pp.65-70.